

Binary Search Trees

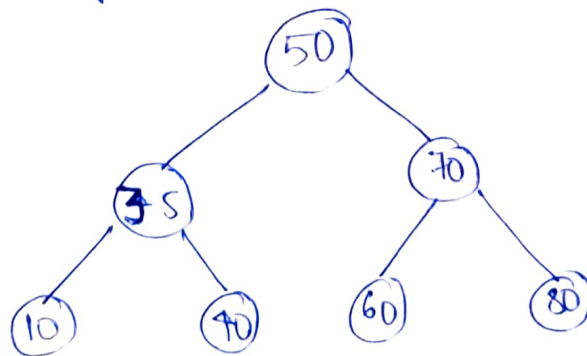
	Array (unsorted)	Array (sorted)	Linked List	BST (balanced)	Hash Table
Search	$O(n)$	$O(\log n)$	$O(n)$	$O(\log n)$	$O(1)$
Insert	$O(1)$	$O(n)$	$O(1)$ $O(n)$ in sorted	$O(\log n)$	$O(1)$
Delete	$O(n)$	$O(n)$	$O(n)$	$O(\log n)$	$O(1)$
Find closest	$O(n)$	$O(\log n)$	$O(n)$	$O(\log n)$	$O(1)$
Sorted Traversal	$O(n \log n)$	$O(n)$	$O(n \log n)$ $O(n)$ in already sorted	$O(n)$	$O(n \log n)$

① BST is preferred when we've to find closest i.e. ceil, floor etc.

② Data is organised in such a way so that it reduces more than half.

③ Data is organised on the binary search algo of arrays.

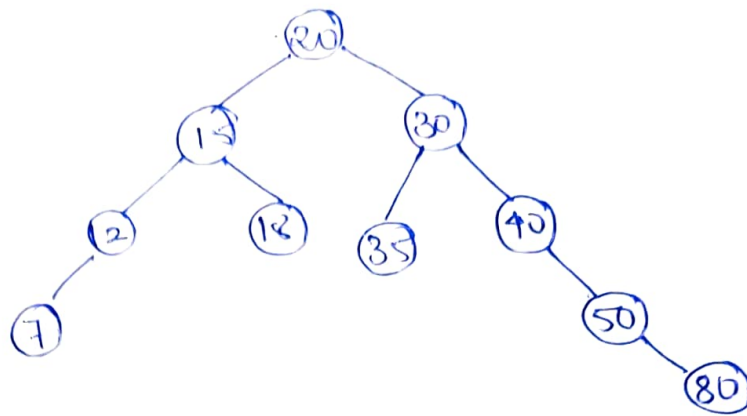
④ keys on the left are smaller and keys on the right are greater.



in C++, implemented as map, set, multimap & multiset

Create a BST

insert 20, 15, 30, 40, 50, 12, 18, 35, 80, 7



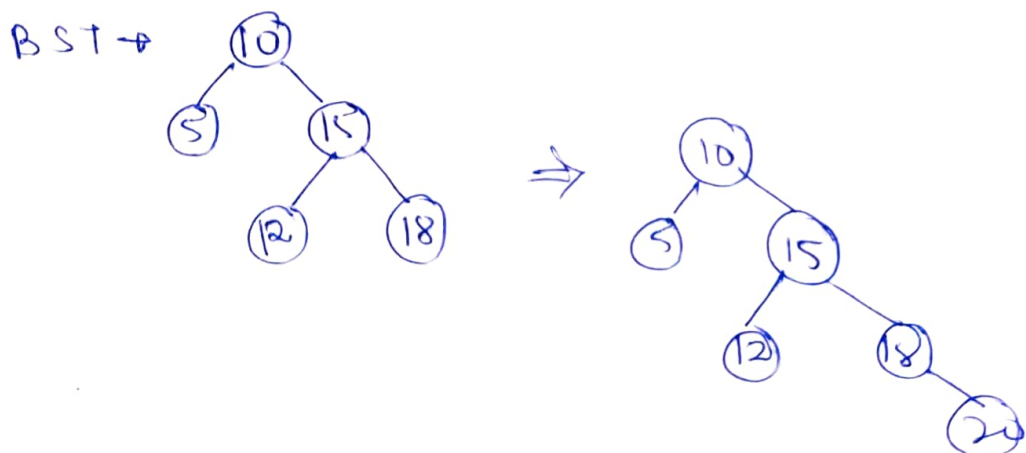
Search in a BST

```
bool searchBST(Node* root, int val) {  
    if (!root) return false;  
    if (root->data == val) return true;  
    if (root->data > val) {  
        return searchBST(root->left, val);  
    }  
    return searchBST(root->right, val);  
}
```

worst case time - $O(n)$
Aux space - $O(n)$.
best case comp $\rightarrow O(\log n)$.

Insert in BST

I/P $\rightarrow x = 20$



node *insert (Node *root, int x) {
 if (root == NULL)
 return new Node(x);

if (root->data > x)

root->left = insert (root->left, x);

else if (root->data < x)

root->right = insert (root->right, x);

return root;

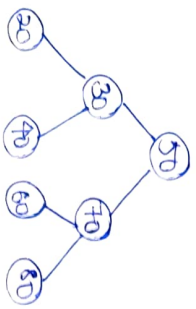
}

Iterative

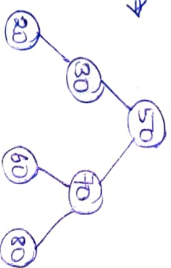
→ first search the node, keep track of parent and join the node to the parent.

Deletion of Node in BST

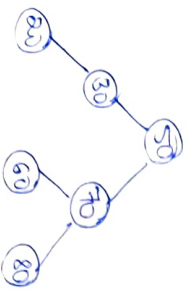
I/P → 40



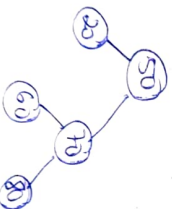
O/P →



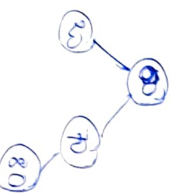
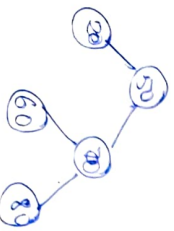
I/P → x = 30



O/P →



I/P → x = 50



If the root is deleted it can be replaced by its left child.

If the root is to be deleted, it can be replaced by its left child or its right child.

③ You can make a rule that either you'll pick the greatest value or the smallest value.

* As the inorder traversal of BST is always sorted,

→ if we pick the smallest element → inorder predecessor
 → greatest element → inorder successor.

There can be three conditions

① both child are null & deleted node is the leaf node. → delete node and return NULL

② both child is NULL → delete node & return NULL

③ both child are not NULL

→ get the successor of the root
 → left most child of the right subtree

root->key = succ->key
 and we will delete the succ->key.

Algorithm

① getting the successor node:-

Node *getSuccessor (Node *curr) {

curr = curr->right;

while (curr->left != NULL && curr->left->right == NULL)

curr = curr->left;

return curr;

}

Node * delNode (Node * root, int x) {

if (!root) return root;

if (root->key > x)

root->left = delNode (root->left, x);

else if (root->key < x)

root->right = delNode (root->right, x);

else {

if (root->left == NULL) {

Node * temp = root->right;

delete root;

return temp;

}

else if (root->right == NULL) {

Node * temp = root->left;

delete root;

return temp;

}

else {

Node * succ = getSuccessor (root);

root->key = succ->key;

root->right = delNode (root->right, succ->key);

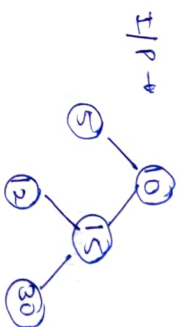
return root;

}

Time complexity = $O(n)$

Floor of BST

• closest smaller value.



I/P →

x = 14

→

Node - 12

same tree

I/P → x = 4

O/P → NULL.

I/P → x = 30

O/P → 30.

Naive solution

Traverse the whole tree and maintain closest smaller.

Efficient solution

Use the concept of binary search.

Node * floor (Node * root, int x) {

Node * res = NULL;

while (!root) {

if (root->key == x) {

return root;

}

if (root->key > x)

root = root->left;

else {

res = root;

root = root->right;

}

return res;

}

ceil of BST

→ greater than or equal to given key (least).

Node *getceil(Node *root, int x) {

if (!root) return NULL; *res = NULL;

while (root) {

if (root->key == x)

return root;

if (root->key < x) {

root = root->right;

res = getceil(root->right);

} else {

res = root;

root = root->left;

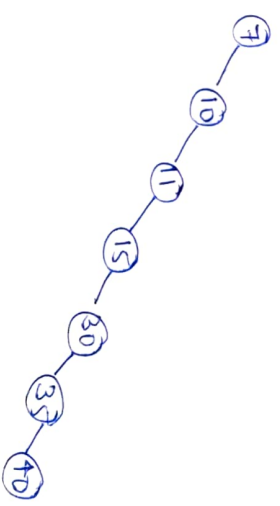
} return res;

}

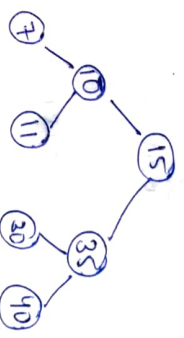
keep the height as $O(\log n)$.

Example

Order-1 - 7, 10, 11, 15, 30, 35, 40.

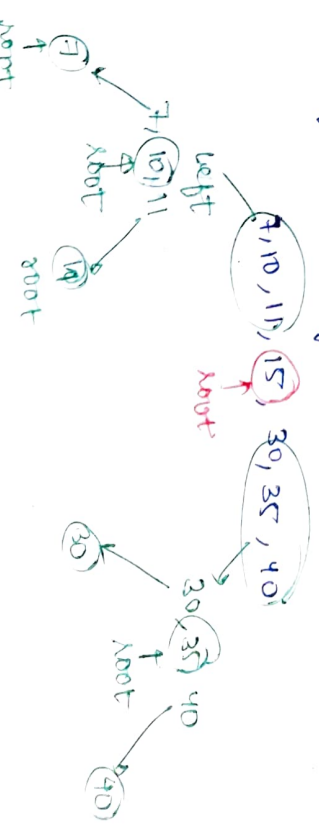


Order-2 - 15, 10, 7, 11, 35, 30, 40



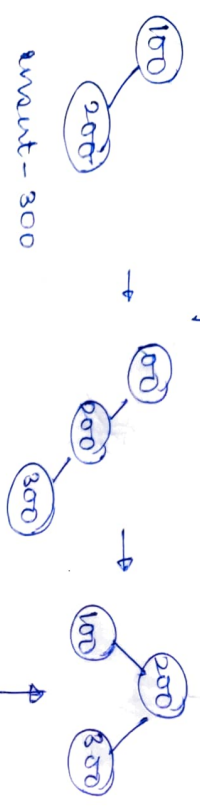
→ height = $\log n$

- ① height and structure is decided by height
- ② if the keys are known prior, we can sort the key and select the middle key as root, and recursively do the same for left and right subtrees



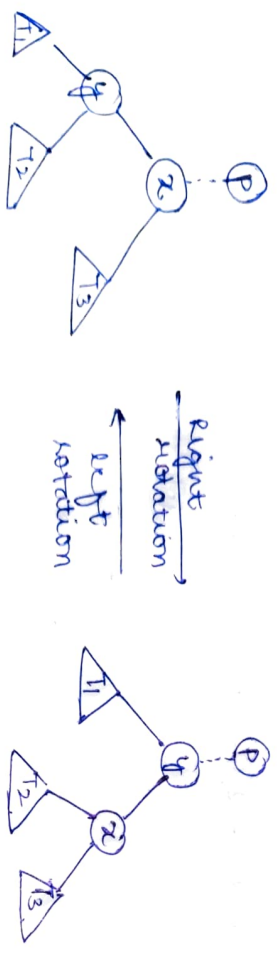
How to maintain height when insert is unbalancing in random way.

→ Do some restructuring



restructuring
(Rotated left)

Rotation



Self Balancing

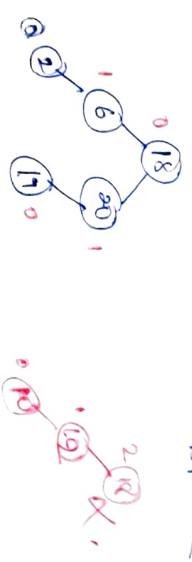
→ AVL Tree (every strict in terms of height)
→ Red black tree

AVL Tree

① For every node the difference of left subtree and right subtree should not exceed 1

Balance factor = |left - right|

BF ≤ 1

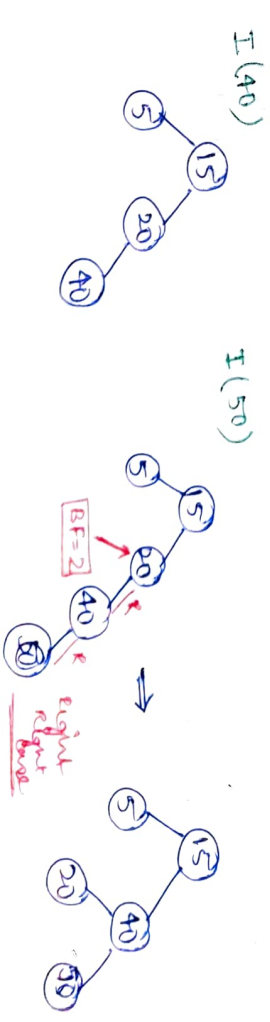
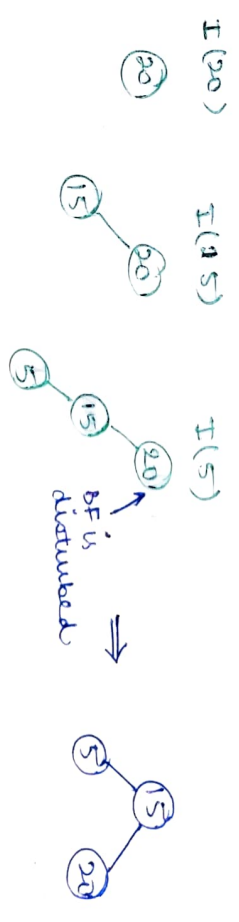


Insert operation

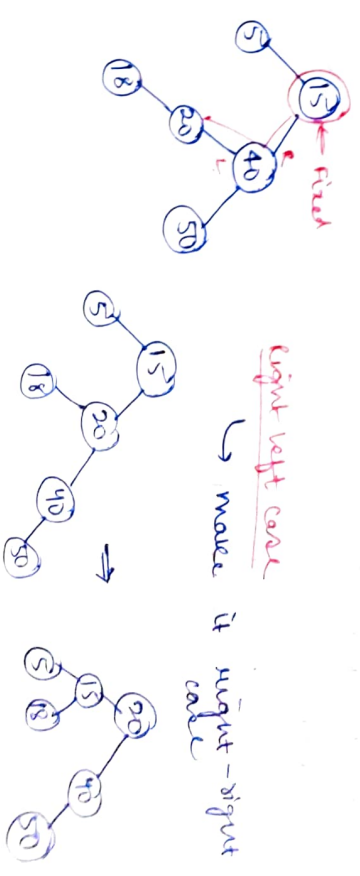
→ Perform normal BST insert.
→ Traverse all ancestors of newly inserted node from node to root
→ If find an unbalanced node, check for any of the below cases

- ① left left
 - ② Right right
 - ③ left right
 - ④ right left
- single rotation
double rotation

Order - 20, 15, 5, 40, 50, 18



right left case
→ make it right-right case



time complexity

insertion - $O(\log n) + O(\log n)$

↑
insertion

↑
travelling to
top to find
unbalanced
ancestor

$$h < c \log (n+2) + b$$

$$c \approx 1.4405$$

$$b \approx -1.3277$$

Red Black BST

→ every node is either black or red

→ Root is always black

→ No two consecutive reds.

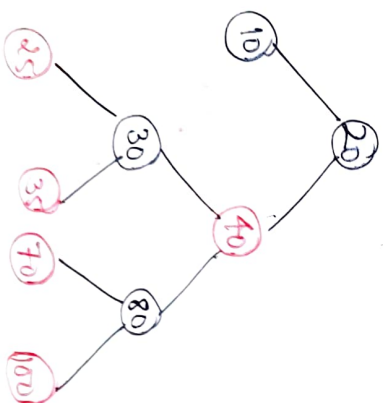
→ Number of black nodes from every node to all of its descendants leaves should be same

→ More as compared to AVL tree, and have less complex insert and delete operation

→ Search is more complex.

→ Number of nodes to the leaves to the farthest descendant can be almost twice the number of nodes in the path to the closest leaf descendant.

• Insert and deletion are less complex and also in insertion
① try rebalancing
② rotation



Applications of BST

- 1) To maintain sorted stream of data
- 2) To implement doubly ended priority queue.
- 3) To solve problems like:

- (a) count smaller/greater in a stream
- (b) First/ceil/greater/smaller in a stream

Set in C++

→ maintain data in sorted order (increasing).
→ Duplicates are not allowed.

insert() → insert element in a set and maintain a sorted order

begin() → points to first element of set

end() → points to the location beyond last element

rbegin() & rend() → Reverse iterators

find() → Search an element

auto it = s.find();
if (it == s.end()) → not found
else → found

clear() → Remove all elements

count() → Returns 0/1 (as count of any element ≤ 1), can remove

For a single element
For a range of elements.

s.erase(it, it2)

not inclusive

lower-bound() →

auto it = s.lower_bound(x)

- ↳ gives an iterator to element if present
- ↳ if not present, give the element just greater than that
- ↳ if there is no element greater than x it returns s.end()

upper-bound() →

auto it = s.upper_bound(x)

- ↳ if x is present, it returns iterator to next element
- ↳ if x is not present - it returns iterator to the next greater element
- ↳ greater than greatest ~~is not~~ s.end()

Internally

↳ set uses Red Black Tree

insert(), find()
count(), lower_bound()
upper_bound(), erase(x)

> $\log(n)$

erase(it) — amortized, $O(1)$

Map in C++ (Ordered)

- ↳ built using red black tree
- ↳ store a key-value pair
- ↳ element are ordered on the basis of key

insert() → inserts element into the map

m.insert({10, 20});

operator() →

m[10] = 20;

if we access something which is not present in map.

cout << m[20];

then it will insert a value to the map
key = 20
value = default int = 0

(20, 0) → inserted

at() → m.at(10) = 300 (updating)

m.at(20)

↳ if 20 is not present, it will throw an exception

m.first() → key
m.second() → values.

upper_bound() → same as set
lower_bound() → same as set.

erase → can pass a key
can pass iterator
can pass range, it, to it2.

Ceiling on left side

I/P = arr[] = {2, 8, 30, 15, 25, 12, 30}

O/P $\Rightarrow$ {-1, -1, -1, 30, 30, 25}

these elements $\geq 15, 25, 30$

smaller element than

I/P = arr[] = {30, 20, 10}

-1, 30, 20

I/P = {10, 20, 30}

-1, -1, -1

Approach

Empty self balancing binary tree

Insert arr[] into s

print t-1 $\leftarrow$ for 1st element (fixed).

for (int i = 1; i < n; i++) {

if (s contains a ceiling of arr[i]) {

print the ceiling

else

print -1

Insert arr[i] into s

}

ceil + lower bound

Kth Smallest Element

Design a DS such that insert, delete, search & find smallest efficiently.

Naive approach

use a BST and traverse in-order and maintain a count. if count == k, return root $\rightarrow$ key.

Efficient approach

To change the structure of BST, augmented BST

class Node {

int x; int count;

Node (x) {

this->x = x;

... pointer init

count = 0;

}

compare (count+1) with k.

1) if same, return root.

2) if greater, recur for left subtree.

3) if smaller, recur for right subtree with

k as (k - (count+1))

skipping left nodes

skipping the right left subtree + node

How to maintain count

1) insert something in left, increment the count

2) while inserting on right, if to count remains unchanged.

Node * insert (Node * root, int x) {

if (root == NULL)

return new Node (x);

if (root->key > x) {

root->left = insert (root->left, x);

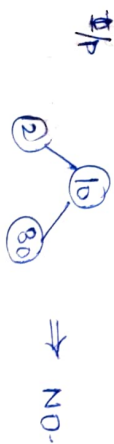
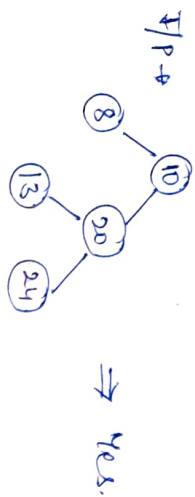
root->count++;

}

else if (x > root->key) root->right =

insert (root->right, x);

check if the binary tree is BST



Naïve solution (Super Efficient) : D

Do inorder traversal of the tree, if the res array is sorted, given tree is BST. (store prev **)

2nd solution

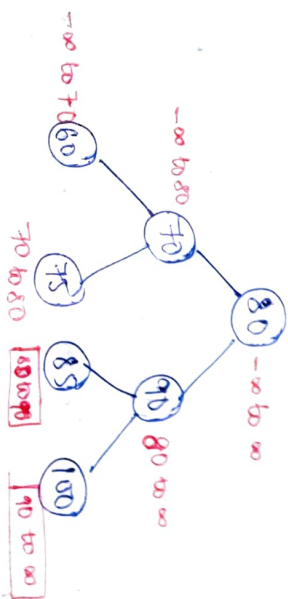
calculate max on left subtree and ⇒ left
min on right subtree ⇒ right

left < root < right

$O(n^2)$

Efficient solution

⇒ pass range with every node.
⇒ for root, range is $-\infty$ to $+\infty$.
⇒ for left child → range upper bound = root → key
for right child → range lower bound = root → key



bool isBST(Node *root, int min, int max) {

if (root == NULL) return true;

return

(root → key > min &&

root → key < max &&

isBST(root → left, min, root → key)

&& isBST(root → right, root → key, max))

isBST (root, INT_MIN, INT_MAX);

$O(n)$

Implementation of 1st solution

int prev = INT_MIN;

bool isBST (Node *root) {

if (root == NULL) return true;

if (!isBST (root → left) == false) return false

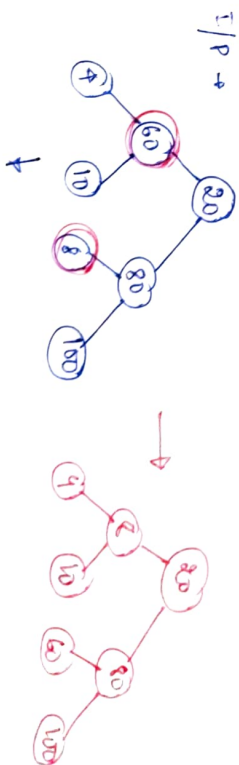
if (root → key ≤ prev) return false;

prev = root → key.

return isBST (root → right);

}

Fix a BST with two nodes swapped



4 60 10 20 80 100

case-1

4	60	10	20	8	80	100
---	----	----	----	---	----	-----

not adjacent

case-2

4	2	10	60	20	80	100
---	---	----	----	----	----	-----

adjacent

• we will update first only one-time

prev = INT_MIN, first = NIL, second = NIL

for (int i=0; i<n; i++) {

if (arr[i] < prev) {

if (first == NIL)

first = prev;

second = arr[i];

}

prev = arr[i];

}

only set one
time

multiple
time
updates.

Node *prev = NULL, *first = NULL, *sec = NULL;

void fixBST (Node *root) {

if (root == NULL) return;

fixBST(root->left);

if (prev != NULL && root->key < prev->key)

if (first == NULL)

first = prev;

second = root;

}

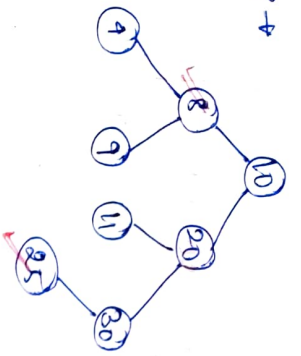
prev = root;

fixBST(root->right);

}

Pair with given sum in BST

I/P →



sum = 33

Yes

Naive solution

- Do an inorder traversal of the tree
- get the sorted array
- Use two pointer approach to get sum

time → $O(n) + O(n)$
 $= O(n)$

Aux → $O(n)$

Efficient solution

- While traversing through the tree, keep adding the nodes in the hashtable if (sum - root-key) is present in hashtable return true.

bool isPairSum(Node *root, int sum,

unordered_set < int > s)

if (root == NULL) return false;

if (pairSum (root → left, sum, s) == true)

return true;

if (s.find (sum - root → key) != s.end())

return true;

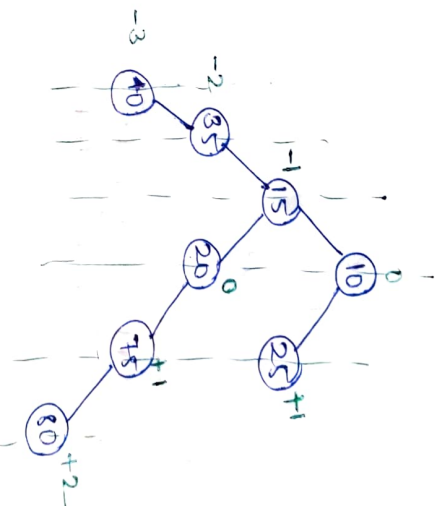
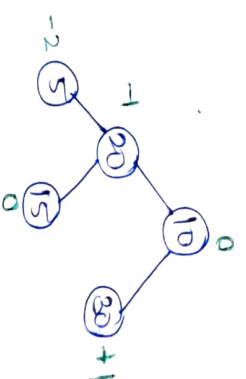
else s.insert (root → key);

return isPairSum (root → right, sum, s);

Vertical sum in a Binary Tree

↳ sum of elements having same horizontal distance

right child = +1
 left child = -1



-3	-2	-1	0	+1	+2
40	35	15	30	15	80

solution

use a map (ordered)

map < int, int > m;

void vsum (Node *root, int ind, map < int, int > &m)

if (root == NULL) return;

vsum (root → left, ind - 1, m);

m[ind] += root → key;

vsum (root → right, ind + 1, m);

void vsum (Node *root)

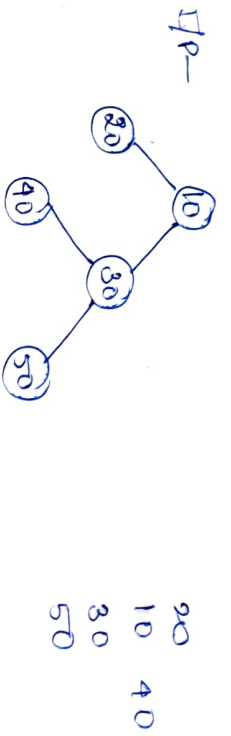
map < int, int > m;

vsum (root, 0, m);

for (auto sum : m)

cout << sum << " ";

Vertical Traversal



• we'll use level order traversal, because we have to maintain order also.
→ we'll use two data structures

- ① queue
→ it will contain address of node.
- ② map <int, vector<int>>.
→ horizontal distance.

void VTraversal (Node *root) {

map<int, vector<int>> mp;
queue<pair<Node*, int>> q;
q.push ({root, 0});
while (q.empty () == false) {

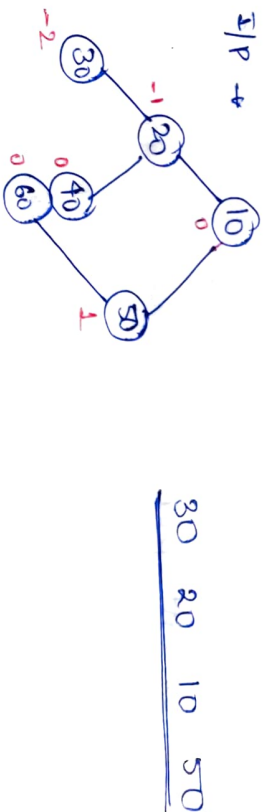
auto p = q.front ();
Node *curr = p.first;
int ind = p.second;

mp[ind].push_back (curr->data);
q.pop ();

if (curr->left) q.push ({curr->left, ind+1});
if (curr->right) q.push ({curr->right, ind+1});

}
print mp
}

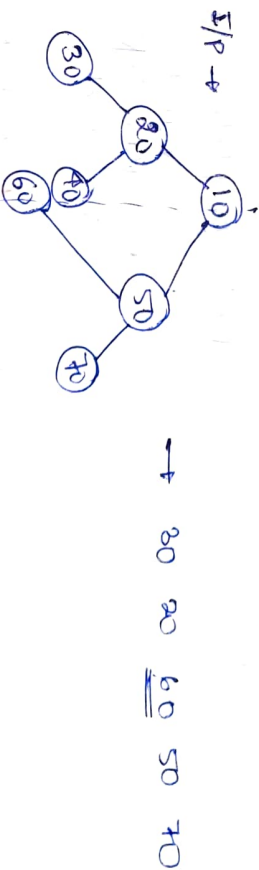
Top View of Binary Tree



→ we'll use the same approach, but this time we'll not push elements instead we'll have only one time iteration for a key (ind) and will not update that.

if (mp.find (ind) == mp.end ())
mp[ind] = curr->key;

Bottom View of Binary Tree



using the same concept, this time we'll update the mp[ind] every time, to get the bottom most node.