

Graph

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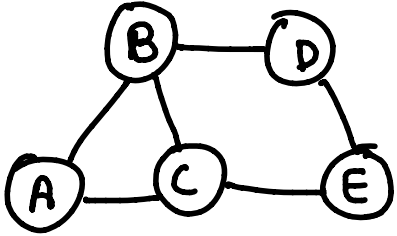
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Graphs

Graph G is a pair (V, E) where V is set of vertices & E is set of edges. $n = |V|$ & $e = |E|$

Eg



$$V = \{A, B, C, D, E\} \quad n = 5$$

$$E = \{AB, AC, BC, BD, CE, DE\} \quad e = 6$$

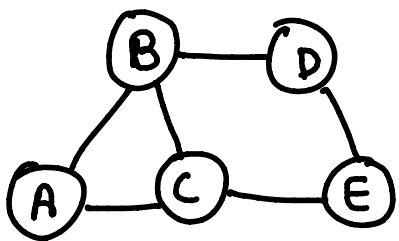
Applications →

Google maps → To find shortest routes

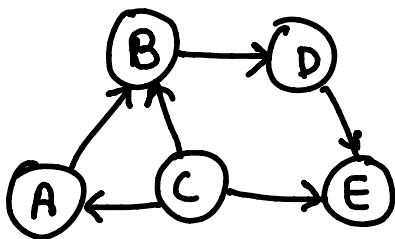
Social network → user, connection
 ↑ ↑
 vertex edge

Types →

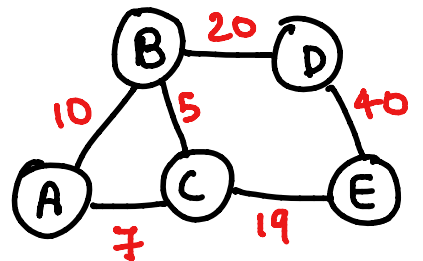
1) Undirected



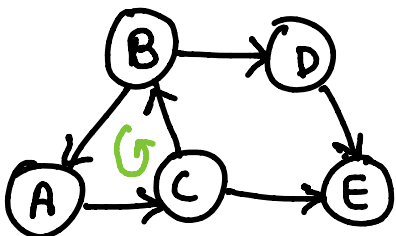
2) Directed



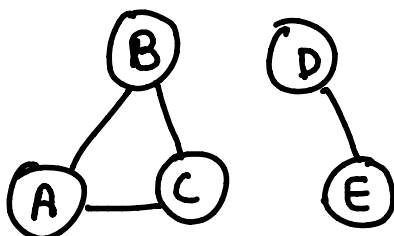
3) Weighted



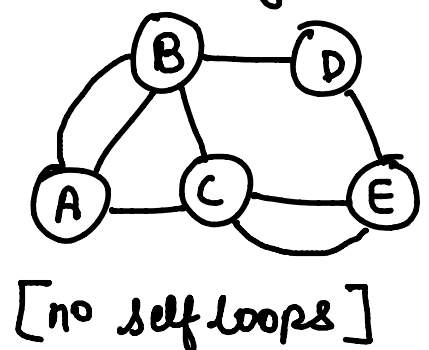
4) Cyclic



5) Disconnected



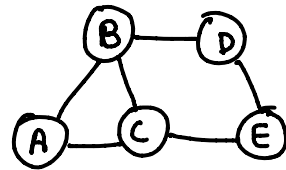
6) Multigraph



Graph Traversal

(a) BFS → visit each and every vertex in a defined order.

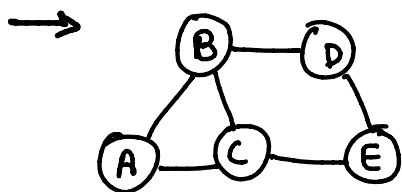
- select node
- visit its unvisited neighbour nodes
- mark it as visited & push into result
- push it into queue
- if no neighbours then pop.
- repeat till queue is empty



queue.

Visited

A	B	C	D	E

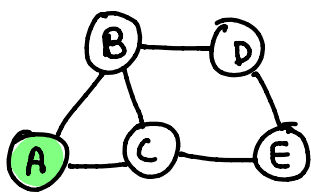


queue.

Visited

A	B	C	D	E

res

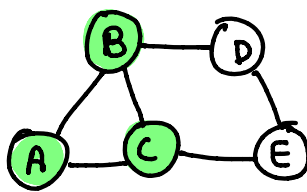


queue.

Visited

A	B	C	D	E

res

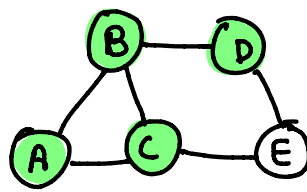


queue.

Visited

A	B	C	D	E

res

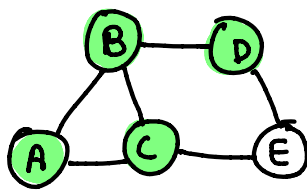


queue.

Visited

A	B	C	D	E

res

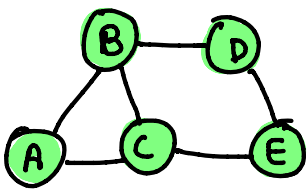


queue.

Visited

A	B	C	D	E

res

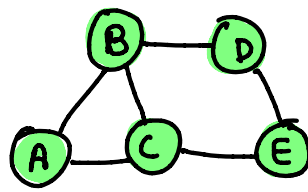


queue.

Visited

A	B	C	D	E

res

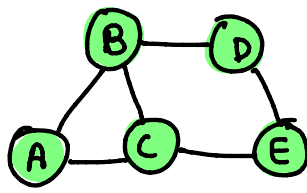


queue.

Visited

A	B	C	D	E

res

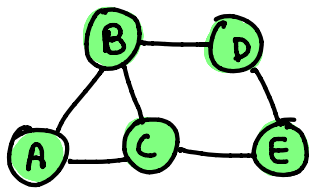


queue.

Visited

A	B	C	D	E

res

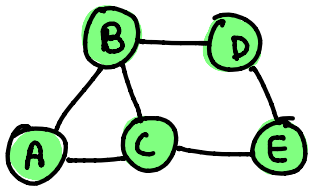


queue.

Visited

A	B	C	D	E

res



queue.

Visited

A	B	C	D	E

res

TC → O(V+E)

SC → O(V)

Return res

Code

```
class Solution {
public:

    vector<int> bfsOfGraph(int V, vector<int> adj[]) {
        vector<int>ans;
        vector<int>vis(V,0);
        queue<int>q;
        q.push(0);

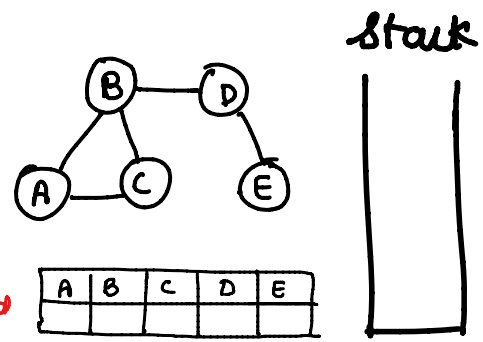
        while(!q.empty()){
            int curr = q.front();
            q.pop();
            vis[curr]=1;
            ans.push_back(curr);
            for(auto it:adj[curr]){
                if(vis[it]==0){
                    vis[it]=1;
                    q.push(it);
                }
            }
        }
        return ans;
    }
};
```

Applications → [BFS]

1. Shortest path
2. Min. spanning tree for unweighted graph
3. Cycle detection
4. GPS
5. Social network.

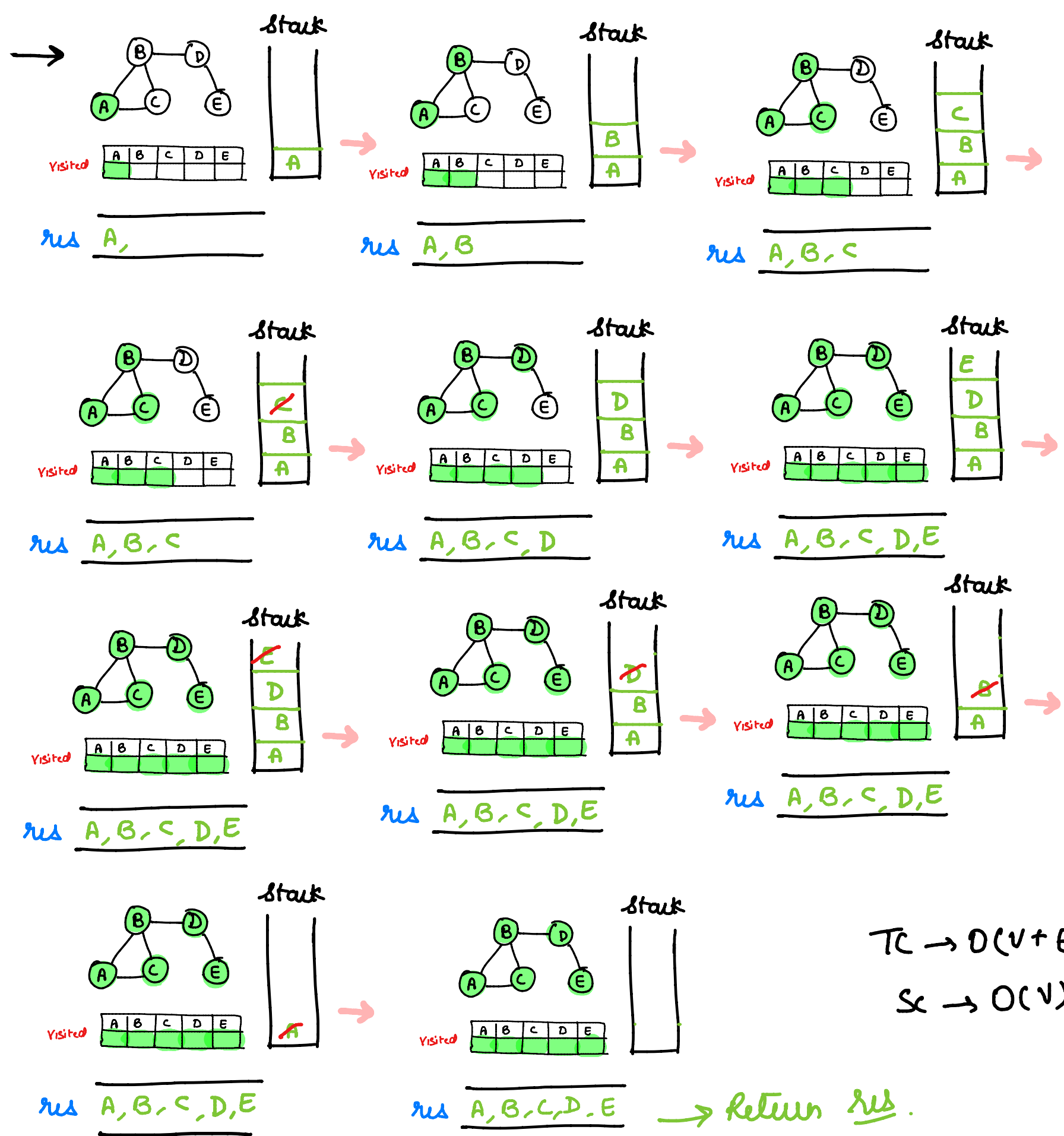
⑥ DFS →

- select node
- visit its unvisited neighbour nodes
- mark it as visited & push into result
- push it into stack
- if no neighbours then pop.
- repeat till stack is empty



Visited

res



TC → O(V + E)
 Sc → O(V)

→ return res.

code

```
class Solution {
public:

    void dfs(vector<int>&ans, vector<int>&vis, int node, vector<int>adj[]){
        vis[node] = 1;
        ans.push_back(node);
        for(auto it:adj[node]){
            if(!vis[it]){
                vis[it] = 1;
                dfs(ans, vis, it, adj);
            }
        }
    }
    vector<int> dfsOfGraph(int V, vector<int> adj[]) {
        vector<int> ans;
        vector<int> vis(V,0);
        for(int i=0; i<V; i++){
            if(vis[i]==0)
                dfs(ans, vis, i, adj);
        }
        return ans;
    }
};
```

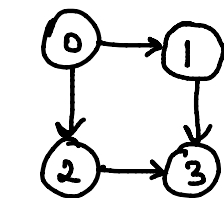
Applications → [DFS]

1. Path finding
2. Cycle detection
3. Topological sort
4. Finding strongly connected components.

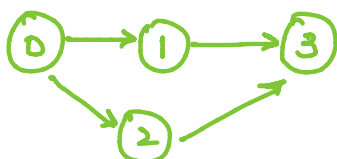
① All paths from src to target

Given a directed acyclic graph, return all paths from node 0 to node $n-1$.

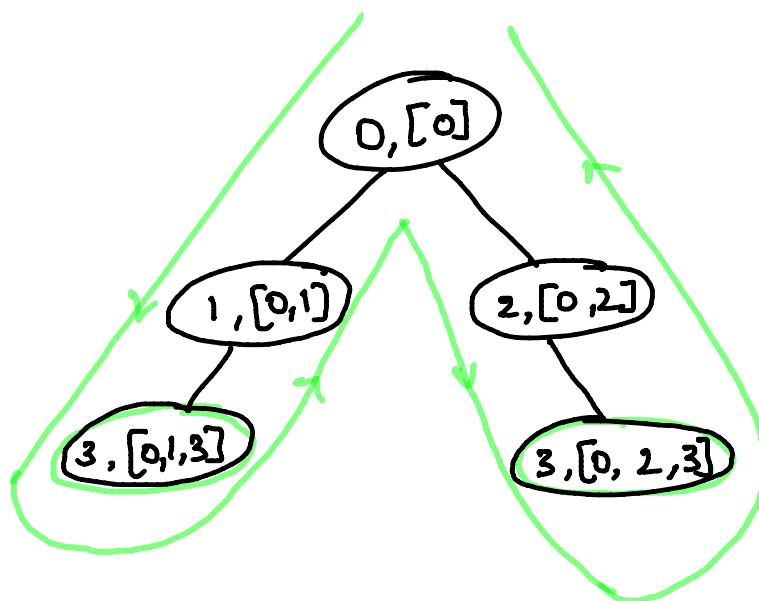
Eg



Path → 1



Path → 2



Code →

TC → $O(V+E)$

V → Vertices

E → edges

SC →

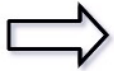
Recursive Stack

+ Result

```
1 class Solution {
2 public:
3     void findAllPaths(vector<vector<int>>&graph, int currNode, vector<bool>&visited,
4                       int n, vector<int> &currPath, vector<vector<int>>&res){
5
6         if(currNode==n-1){
7             res.push_back(currPath);
8             return;
9         }
10
11         if(visited[currNode]==true) return;
12
13         // backtrack for every node
14         visited[currNode] = true;
15
16         for(auto neighbour: graph[currNode]){
17             currPath.push_back(neighbour);
18             findAllPaths(graph, neighbour, visited, n, currPath, res);
19             currPath.pop_back();
20         }
21
22         visited[currNode] = false;
23     }
24
25     vector<vector<int>> allPathsSourceTarget(vector<vector<int>>& graph) {
26         vector<vector<int>> res;
27         vector<int> currPath;
28         int n = graph.size();
29         vector<bool> visited(n);
30
31         // traversing from 0 node
32         currPath.push_back(0);
33
34         findAllPaths(graph, 0, visited, n, currPath, res);
35         return res;
36     }
37 };
```

② Flood Fill → If 0 then blocker, else fill it with given color.

	0	1	2
0	1	1	1
1	1	1	0
2	1	0	1



2	2	2
2	2	0
2	0	1

* Perform flood fill from given index in all 4-directions & the cell should have same color as src

✓ let's follow the order to fill → UP, DOWN, LEFT, RIGHT

Eg In above case starting point is (1,1) & value = 1 so

(*)

1	1	1
1	1	0
1	0	1



1	1	1
1	2	0
1	0	1

up

1	2	1
1	2	0
1	0	1

← from here up is not possible & down is filled so left

left

2	2	1
1	2	0
1	0	1

down

2	2	1
2	2	0
1	0	1

down

2	2	1
2	2	0
2	0	1

→ no direction is possible so return

from here up not possible so down

from here up not possible so down

2	2	1
2	2	0
2	0	1

right

2	2	2
2	2	0
2	0	1

no other way possible

Result

2	2	2
2	2	0
2	0	1

Code

```
1  class Solution {
2  public:
3      void floodFiller(vector<vector<int>>& image, int i, int j,
4      int m, int n, int currColor, int newColor)
5      {
6          if(i<0 || i>=m || j<0 || j>= n || image[i][j] == newColor
7              || image[i][j] != currColor)
8              return;
9
10         image[i][j] = newColor;
11         floodFiller( image, i-1, j, m, n, currColor, newColor);
12         floodFiller( image, i+1, j, m, n, currColor, newColor);
13         floodFiller( image, i, j-1, m, n, currColor, newColor);
14         floodFiller( image, i, j+1, m, n, currColor, newColor);
15     }
16
17     vector<vector<int>> floodFill(vector<vector<int>>& image, int sr,
18     int sc, int newColor)
19     {
20         int m = image.size();
21         int n = image[0].size();
22         int currColor = image[sr][sc];
23         floodFiller(image, sr, sc, m, n, currColor, newColor);
24         return image;
25     }
26 };
```

$T_c \rightarrow O(mn)$

$S_c \rightarrow O(h)$

↳ recursive stack

③ Number of Islands → Given grid of 1 (land) & 0 (water), return no. of islands.

Eg

	0	1	2	3	4
0	1	1	0	0	0
1	1	1	0	0	0
2	0	0	1	0	0
3	0	0	0	1	1

- Always start dfs only if value = 1 & change its value to 0, so it cannot be visited again
- if initial value = 0 then skip
- initially ans = 0

• let's start from (0,0) & try moving U, D, L, R
→ the traversal goes in this order

(0,0) → (1,0) → (1,1) → (0,1) i.e.
& update ans.

1	1	0	0	0
1	1	0	0	0
0	0	1	0	0
0	0	0	1	1

ans = 1.

→ now grid becomes

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	1	0	0
3	0	0	0	1	1

- now, we can skip every entry from (1,0) to (2,1) as they are 0s
- now start from (2,2), as U, D, L, R is not possible, set its value = 0 & update ans.
ans = 2.

→ now grid becomes

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	1	1

- now, we can skip every entry from (2,3) to (3,2) as they are 0s
- now start from (3,3), it goes as follows
(3,3) → (3,4)
- further traversal from (3,4) is not possible

ans = 3. ans = 3

Code

```
1 class Solution {
2 public:
3     void countIsland(vector<vector<char>>& grid, int currRow, int currCol, int row, int col){
4         if(currRow<0 || currRow>=row || currCol<0 || currCol>=col || grid[currRow][currCol]=='0')
5             return;
6
7         grid[currRow][currCol] = '0';
8         countIsland(grid, currRow-1, currCol, row, col);
9         countIsland(grid, currRow+1, currCol, row, col);
10        countIsland(grid, currRow, currCol-1, row, col);
11        countIsland(grid, currRow, currCol+1, row, col);
12    }
13
14    int numIslands(vector<vector<char>>& grid) {
15        int ans = 0;
16        int row = grid.size();
17        int col = grid[0].size();
18
19        for(int currRow = 0; currRow < row; currRow++)
20            for(int currCol = 0; currCol < col; currCol++)
21                if(grid[currRow][currCol]=='1'){
22                    ans++;
23                    countIsland(grid, currRow, currCol, row, col);
24                }
25
26        return ans;
27    }
28 };
```

$T_c \rightarrow O(mn)$ Avg case
 $O(m^2n^2)$ Worst case

④ Max Area of the Island

- * Intuition is same as previous problem.
- * Minor Tweak to count number of 1s in island.
- * Once entire island Traversal is done ,
compare for maxArea of island.

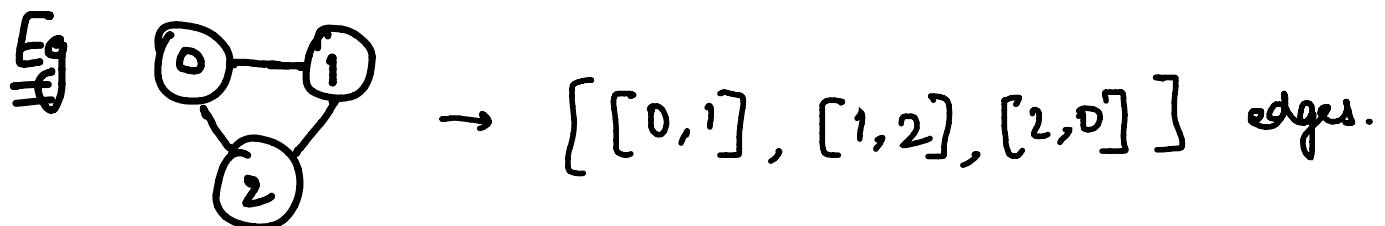
Tc $\rightarrow O(mn)$ Avg case.

code $\rightarrow$

```
1 class Solution {
2 public:
3     int findArea(vector<vector<int>>& grid, int currRow, int currCol, int m, int n){
4         if(currRow<0 || currCol<0 || currRow>=m || currCol>=n || grid[currRow][currCol]==0)
5             return 0;
6
7         grid[currRow][currCol]=0;
8
9         // this is for single cell where we started traversing
10        int count = 1;
11        count += findArea(grid, currRow-1, currCol, m, n);
12        count += findArea(grid, currRow+1, currCol, m, n);
13        count += findArea(grid, currRow, currCol-1, m, n);
14        count += findArea(grid, currRow, currCol+1, m, n);
15        return count;
16    }
17    int maxAreaOfIsland(vector<vector<int>>& grid) {
18        int m = grid.size();
19        int n = grid[0].size();
20        int ans = 0;
21        for(int currRow = 0; currRow<m; currRow++){
22            for(int currCol = 0; currCol<n; currCol++){
23                if(grid[currRow][currCol]==1){
24                    ans = max(ans, findArea(grid, currRow, currCol, m, n));
25                }
26            }
27        }
28        return ans;
29    };
}
```

⑤ Find if path exist in graph.

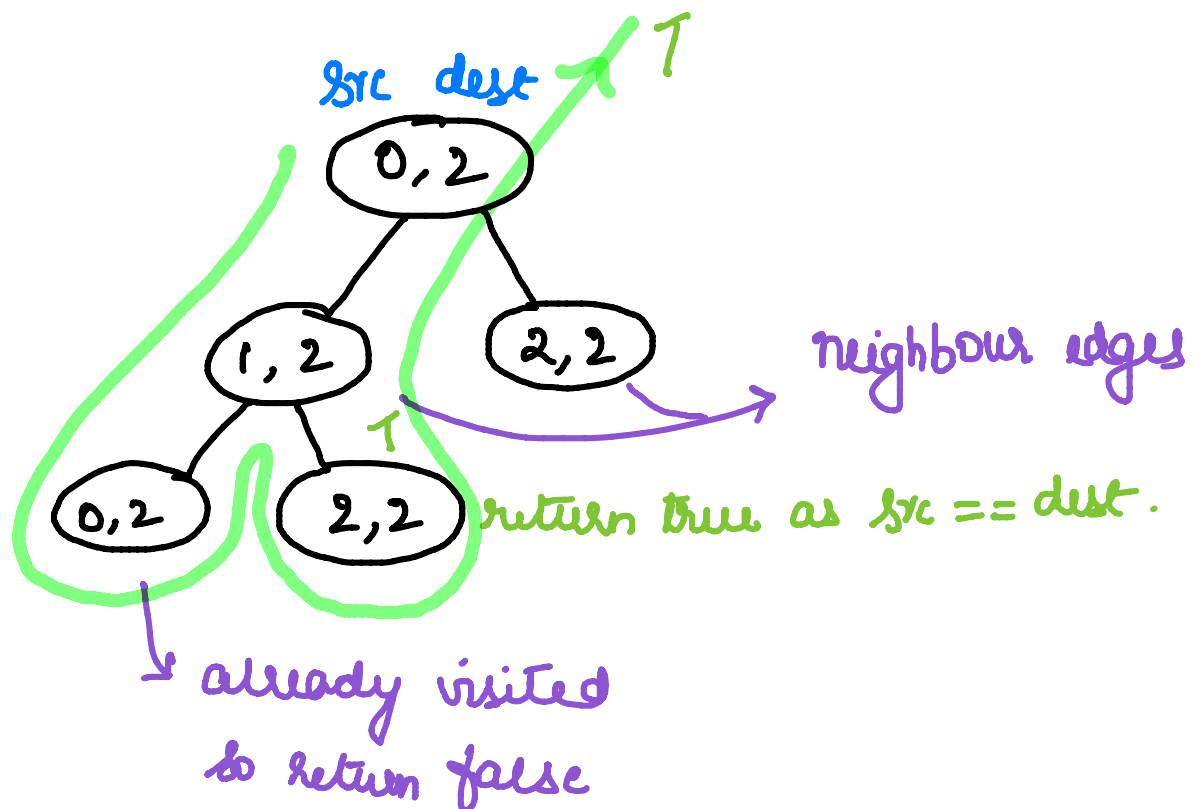
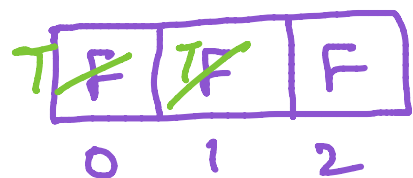
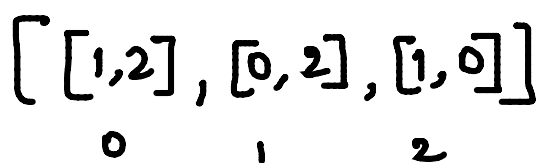
Given src, dest, no. of nodes & set of edges, find if path exist b/w src & dest.



$n = 3$ edges = $[[0, 1], [1, 2], [2, 0]]$ src = 0, dest = 2.

1) Create a graph using adj list rep. $\begin{bmatrix} [1,2], [0,2], [1,0] \end{bmatrix}$

2) Perform dfs

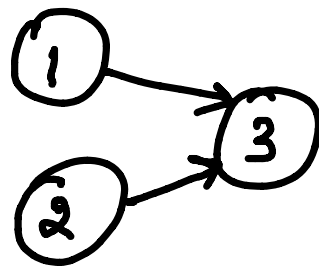


Code →

```
1  class Solution {
2  public:
3      bool validPath(int n, vector<vector<int>>& edges, int src, int dest) {
4
5          vector<vector<int>>graph(n);
6          for(int i=0;i<edges.size();i++)
7          {
8              int v1 = edges[i][0];
9              int v2 = edges[i][1];
10             graph[v1].push_back(v2);
11             graph[v2].push_back(v1);
12
13         }
14         vector<bool>vis(n,false);
15         return pathExist(src, dest, graph, vis);
16     }
17
18     bool pathExist(int src , int dest,vector<vector<int>>&graph,vector<bool>&vis){
19
20         if(src==dest)return true;
21
22         vis[src]=true;
23
24         for(int i=0;i<graph[src].size();i++)
25             if(vis[graph[src][i]]==false)
26                 if(pathExist(graph[src][i],dest,graph,vis)==true)
27                     return true;
28
29         return false;
30     }
31 };
```

⑥ Find the town judge

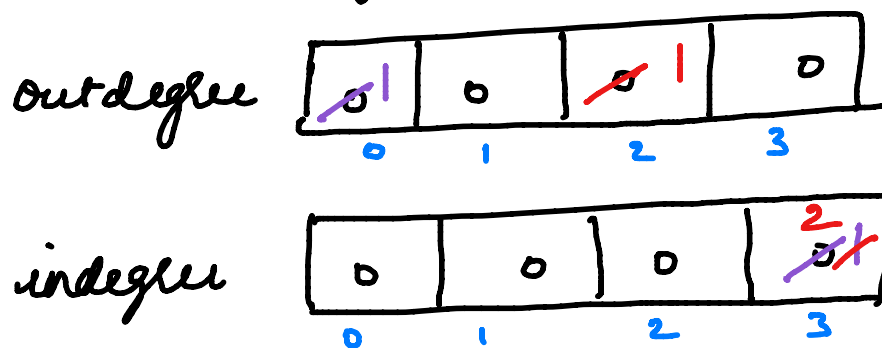
$n = 3$, $trust = [[1, 3], [2, 3]]$ →



* In degree of town judge = $n-1$

& outdegree = 0

✓ Create 2 arrays



for $[1, 3]$

indegree of 1 ↑

outdegree of 3 ↑

for $[2, 3]$

indegree of 2 ↑

outdegree of 3 ↑

→ traverse both indegree & outdegree

if $\text{indegree} == 0$ &&

$\text{outdegree} == n-1$

then return that vertex

code

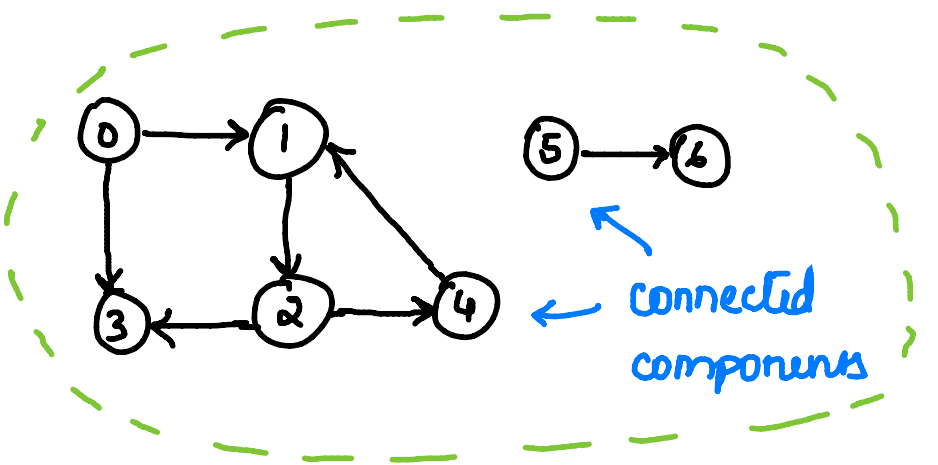
```
1  class Solution {
2  public:
3      int findJudge(int n, vector<vector<int>>& trust) {
4          vector<int> indegree(n+1,0);
5          vector<int> outdegree(n+1,0);
6          for(int i=0;i<trust.size();i++)
7          {
8              int v1 = trust[i][0];
9              int v2 = trust[i][1];
10             outdegree[v1]+=1;
11             indegree[v2]+=1;
12         }
13         for(int i=1;i<=n;i++)
14         {
15             if(outdegree[i]==0 && indegree[i]==n-1)
16                 return i;
17         }
18         return -1;
19     }
20 };
```

⑦ Detect cycle in a directed graph

Consider a graph with 'n' vertices labelled as $[0..n-1]$

Eg $n = 7$ $[0, 1, 2, 3, 4, 5, 6]$

Graph $\rightarrow$



* To detect cycle, check for backedge.

Let's start dfs from 0 vertex.

* At every vertex, check if its already visited, if already visited then check if it is present in recursive stack.

If present, then it indicates back edge $\rightarrow$ Returns True

* If vertex is not visited then mark it in visited array & recursive stack

visited $\rightarrow \{0, 1, 2, 3, 4\}$

Recursive stack $\rightarrow \{0, 1, 2, \cancel{3}, 4\}$

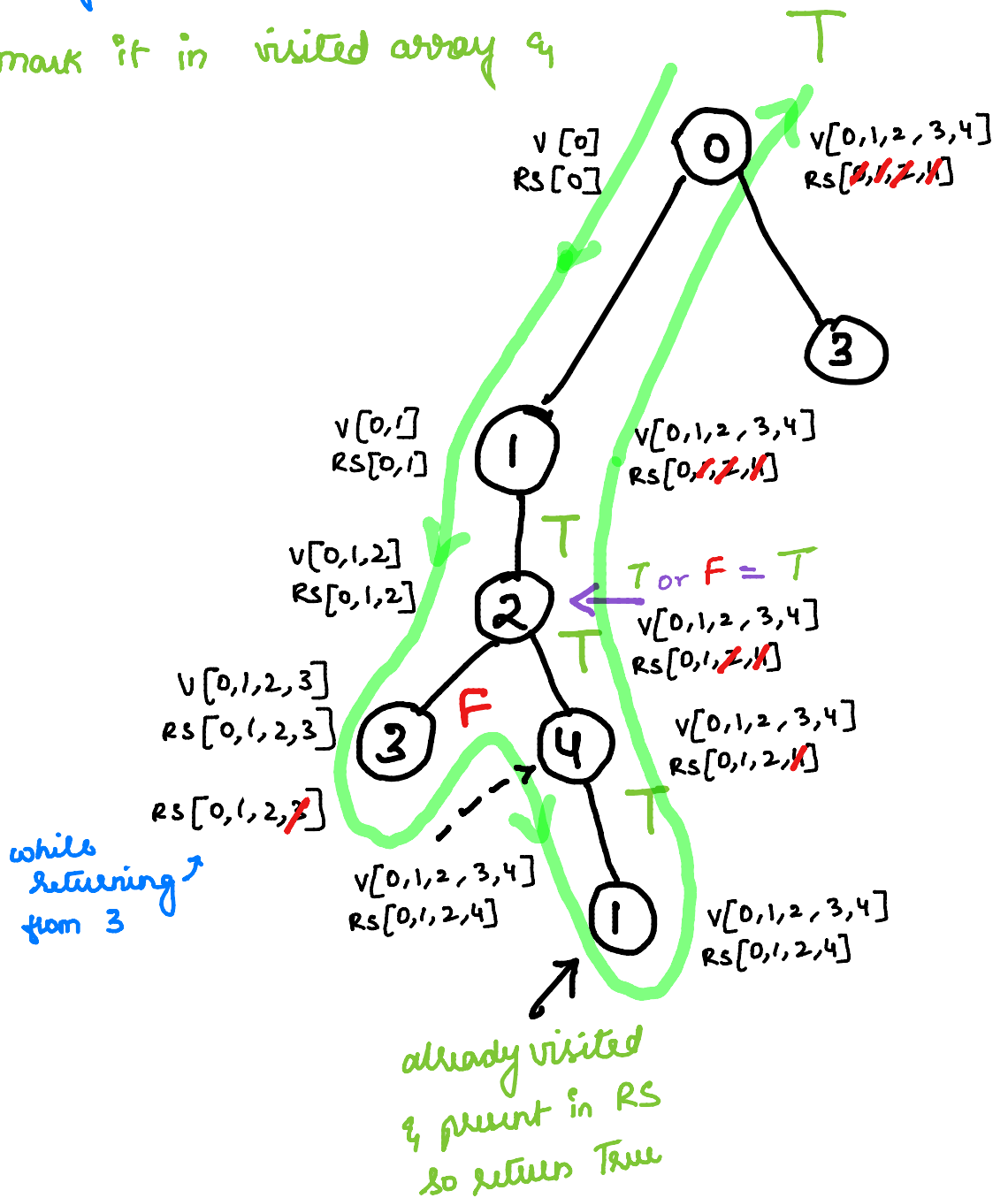
* At 3 vertex, there's no neighbour
 & no cycle is detected so return F.

Before returning, undo change made in Recursive stack by popping it.

visited $\rightarrow \{0, 1, 2, 3, 4, 1\}$ 4

Recursive stack $\rightarrow \{0, 1, 2, \cancel{3}, 4\}$

1 is already present in recursive stack so return true.



Code

$T_c \rightarrow O(V+E)$

$S_c \rightarrow O(V)$

```
1  class Solution {
2      public:
3          bool dfs(int node, vector<int>&vis, vector<int>&rs, vector<int> adj[])
4          {
5              vis[node]=1;
6              rs[node]=1;
7              for(auto it:adj[node])
8              {
9                  if(vis[it]==0){
10                     if(dfs(it,vis,rs,adj))
11                         return true;
12                 }
13                 else if(rs[it]==1)
14                     return true;
15             }
16             rs[node]=0;
17             return false;
18         }
19         bool isCyclic(int V, vector<int> adj[]) {
20
21             vector<int>vis(V,0);
22             vector<int>rs(V,0);
23
24             for(int i=0;i<V;i++)
25             {
26                 if(vis[i]==0)
27                     if(dfs(i,vis,rs,adj))
28                         return true;
29             }
30             return false;
31         }
32     };
```

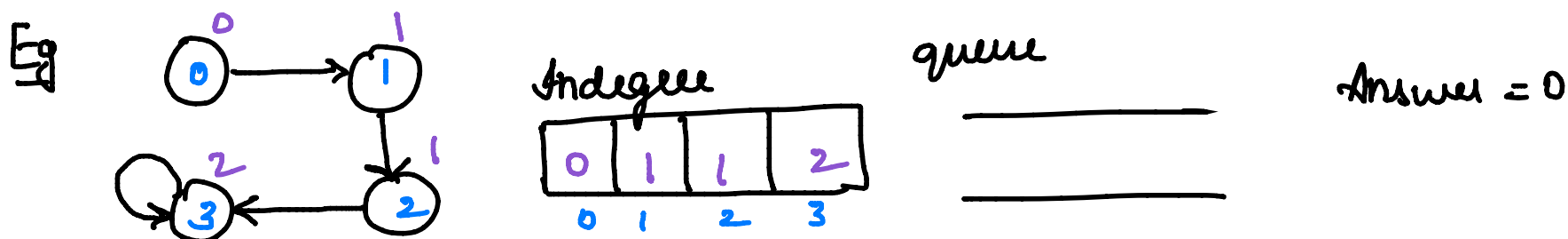
★ Kahn's Algorithm → To find topological Ordering

↓
can be used to find cycle using BFS.

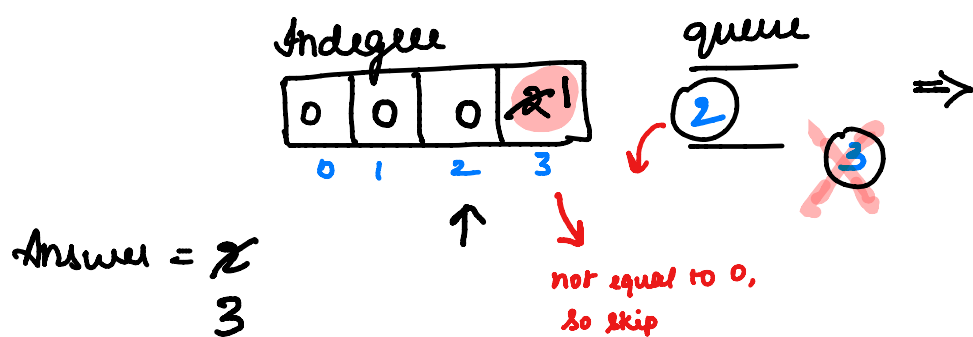
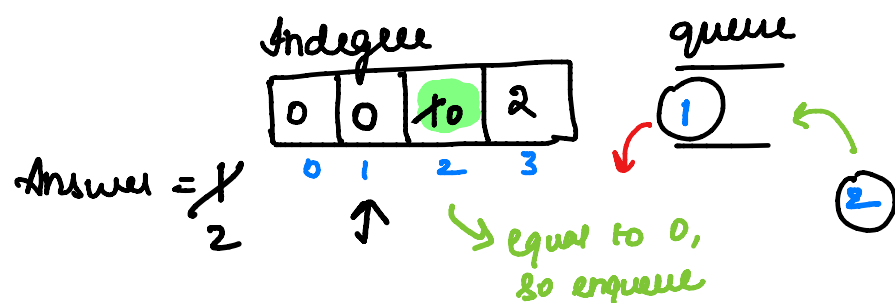
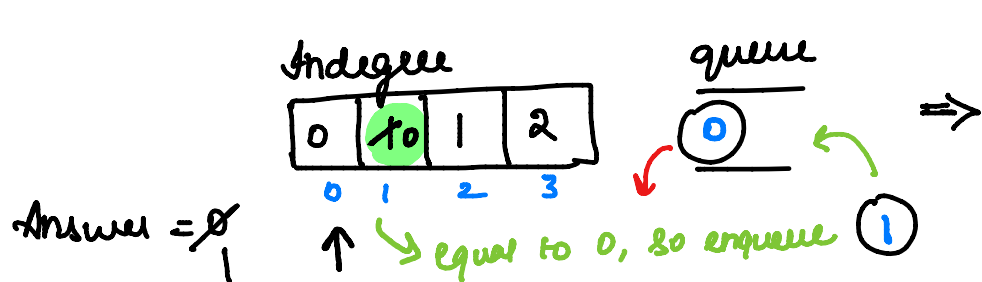
① Find indegree of every vertex in graph & answer = 0

② If indegree of vertex is 0, then push into queue & do bfs till queue is not empty & while doing bfs decrease the indegree of neighbour by 1.
if indegree of neighbour = 0, then enqueue & increment answer by 1

③ If answer $\neq$ no. of vertices then cycle is present.



→ As indegree of 0 is 0, we push into queue & do bfs till queue is not empty.



Answer = 3
No of vertices = 4
∴ cycle present.

code

```
1  class Solution{
2      public:
3          bool isCyclic(int V, vector<int> adj[]) {
4
5              vector<int> indegree(V,0);
6              for (int i = 0; i < V; i++)
7                  for(int it : adj[i])
8                      indegree[it]++;
9
10             queue<int> q;
11             int ans = 0;
12             unordered_set<int> vis;
13
14             for (int i=0; i<V; i++)
15             {
16                 if(indegree[i]==0){
17                     q.push(i);
18                     ans+=1;
19                 }
20             }
21
22             while(!q.empty())
23             {
24                 int currvertex = q.front();
25                 q.pop();
26                 if(vis.find(currvertex)!=vis.end())
27                     continue;
28                 vis.insert(currvertex);
29                 for(int neighbour:adj[currvertex])
30                 {
31                     indegree[neighbour]-=1;
32                     if(indegree[neighbour]==0)
33                     {
34                         q.push(neighbour);
35                         ans+=1;
36                     }
37                 }
38             }
39             if(ans==V) return false;
40             return true;
41         }
42     };
```

⑧ Topological sort

→ use Kahn's algorithm. & add node to results while performing dfs.

Code →

$TC \rightarrow O(V + E)$

$SC \rightarrow O(V)$

```
1  class Solution
2  {
3      public:
4      vector<int> topoSort(int V, vector<int> adj[])
5      {
6          vector<int> indegree(V,0) ,res;
7
8          for(int i=0; i<V; i++)
9              for(auto it:adj[i])
10                 indegree[it]++;
11
12         queue<int> q;
13         int ans = 0;
14         unordered_set<int> vis;
15
16         for(int i=0; i<V; i++)
17         {
18             if(indegree[i]==0){
19                 q.push(i);
20                 ans+=1;
21             }
22         }
23
24         while(!q.empty())
25         {
26             int curr = q.front();
27             q.pop();
28
29             // add to res
30             res.push_back(curr);
31
32             if(vis.find(curr)!=vis.end())
33                 continue;
34
35             vis.insert(curr);
36
37             for(int neighbour: adj[curr])
38             {
39                 indegree[neighbour]-=1;
40                 if(indegree[neighbour]==0)
41                 {
42                     q.push(neighbour);
43                     ans+=1;
44                 }
45             }
46         }
47
48         return res;
49     }
50 };
```

⑨ Course Schedule → can be solved using Kahn's algo.

$$Tc \rightarrow O(V + E)$$

$$Sc \rightarrow O(V + E)$$

code →

```
1  class Solution {
2  public:
3      vector<vector<int>> createGraph(int n, vector<vector<int>>& pre){
4          vector<vector<int>> graph(n);
5          for(auto it:pre){
6              int v = it[1];
7              int u = it[0];
8              graph[v].push_back(u);
9          }
10         return graph;
11     }
12
13     bool canFinish(int n, vector<vector<int>>& pre) {
14         vector<vector<int>> graph = createGraph(n, pre);
15         vector<int> indegree(n, 0);
16         for(int i=0; i<n; i++)
17             for(int it: graph[i])
18                 indegree[it]++;
19
20         queue<int> q;
21         int ans = 0;
22         unordered_set<int> vis;
23
24         for(int i=0; i<n; i++)
25             if(indegree[i]==0){
26                 q.push(i);
27                 ans++;
28             }
29
30         while(!q.empty()){
31             int currvertex = q.front();
32             q.pop();
33             if(vis.find(currvertex)!=vis.end())
34                 continue;
35             vis.insert(currvertex);
36             for(int neighbour: graph[currvertex]){
37                 indegree[neighbour]--;
38                 if(indegree[neighbour]==0){
39                     q.push(neighbour);
40                     ans++;
41                 }
42             }
43         }
44         if(ans==n) return true;
45         return false;
46     }
47 };
```

10) Course Schedule - II

$n \rightarrow$ no. of courses [vertices]

Topological sort only for DAG

$pru \rightarrow$ edges $[v, u]$

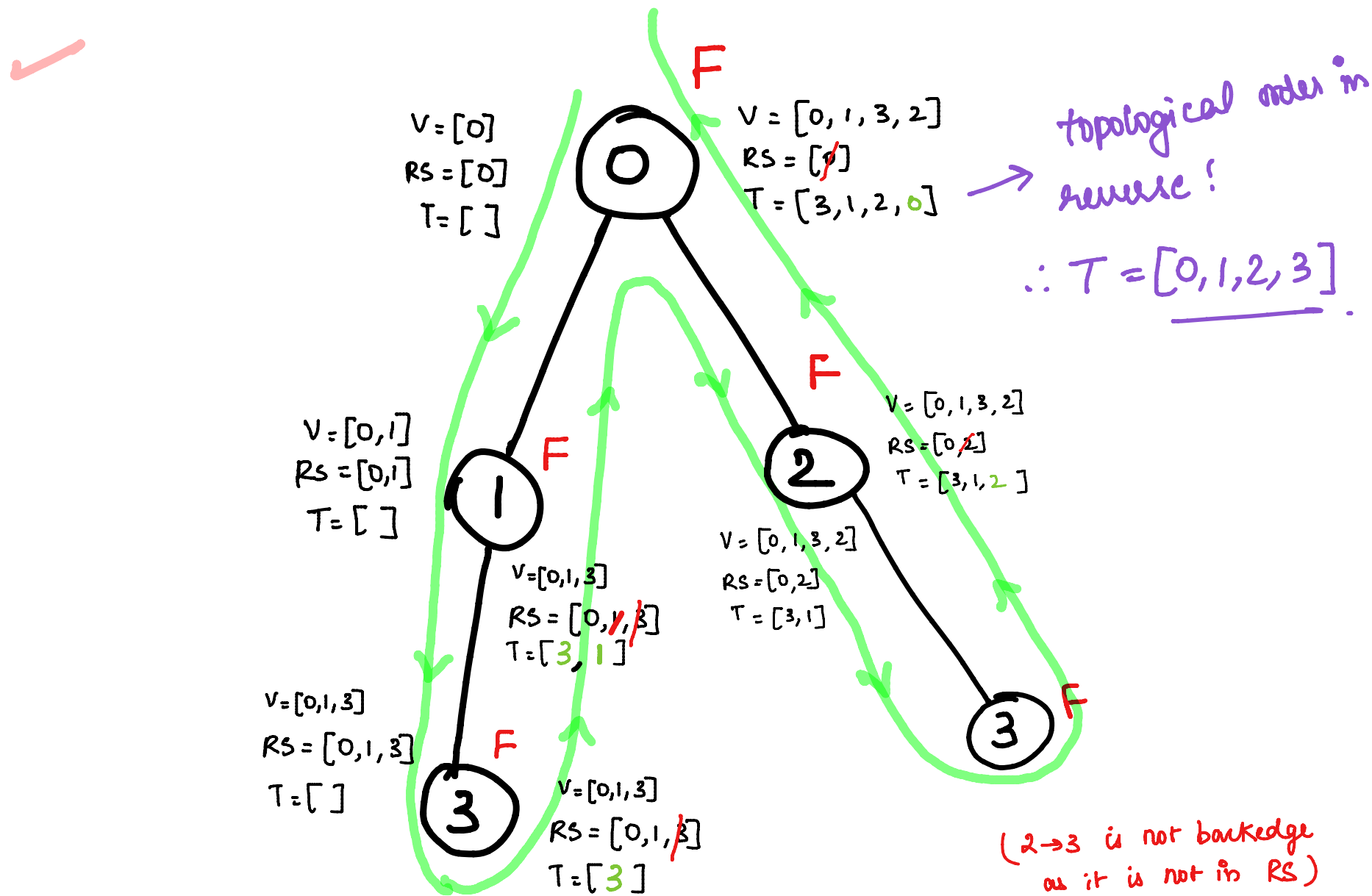
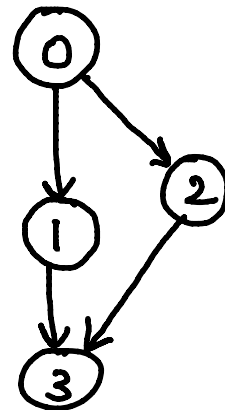
"u should be completed before v"

Eg $n \rightarrow 4$ (0, 1, 2, 3)

$pru \rightarrow [[1, 0], [2, 0], [3, 1], [3, 2]]$

Initially

$V = [], RS = [], traversal = []$



while returning from 3 ↑
pop 3 & push into traversal array.
return F, as no cycle is found

Code →

$T_c \rightarrow O(V + E)$

$S_c \rightarrow O(V + E)$

```
1  class Solution {
2  public:
3      bool dfs(vector<vector<int>>&graph, int i, vector<int> &vis,
4              vector<int> &rs, vector<int> &traversal){
5
6          vis[i] = 1;
7          rs[i] = 1;
8          for(int neighbour: graph[i]){
9              if(vis[neighbour]==0){
10                 if(dfs(graph, neighbour, vis, rs, traversal))
11                     return true;
12             }
13             else if(rs[neighbour]==1) return true;
14         }
15         traversal.push_back(i);
16         rs[i]=0;
17         return false;
18     }
19
20     vector<vector<int>> createGraph(int n, vector<vector<int>>& pre){
21         vector<vector<int>> graph(n);
22         for(auto it:pre){
23             int v = it[1];
24             int u = it[0];
25             graph[v].push_back(u);
26         }
27         return graph;
28     }
29
30     vector<int> findOrder(int n, vector<vector<int>>& pre) {
31         vector<vector<int>> graph = createGraph(n, pre);
32         vector<int> vis(n,0), rs(n,0), traversal;
33         for(int i=0; i<n; i++){
34             if(vis[i]==0)
35                 if(dfs(graph, i, vis, rs, traversal)) return {};
36         }
37         reverse(traversal.begin(), traversal.end());
38         return traversal;
39     }
40 };
```

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